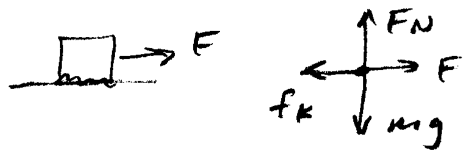
2nd Law problems - with friction

1) No acceleration



$\Sigma \vec{F} = m\vec{a}$

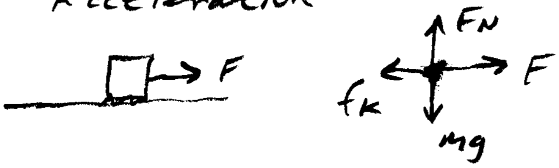
$F - f_k = 0$

$F - \mu_k mg = 0$

$\boxed{F = \mu_k mg}$

$a = 0$

2) Acceleration



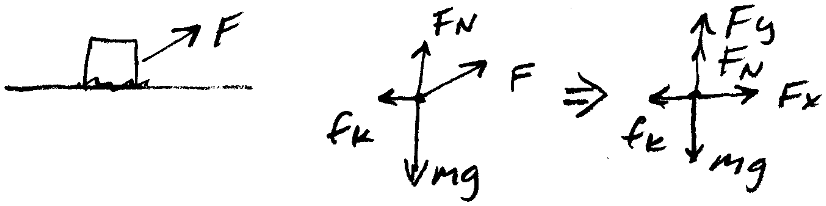
$\Sigma \vec{F} = m\vec{a}$

$F - f_k = ma$

$F - \mu_k mg = ma$

$\boxed{a = \dfrac{F - \mu_k mg}{m}}$

3) Box pulled at angle - acceleration



$\Sigma \vec{F}_y = m\vec{a}_y^{\,0}$

$F_y + F_N - mg = 0$

$F_N = -F_y + mg$

$F_N = mg - F\sin\theta$

$\Sigma \vec{F}_x = m\vec{a}_x$

$F_x - f_k = ma_x$

$F\cos\theta - \mu_k F_N = ma_x$

$F\cos\theta - \mu_k(mg - F\sin\theta) = ma_x$

$\boxed{a_x = \dfrac{F\cos\theta - \mu_k(mg - F\sin\theta)}{m}}$

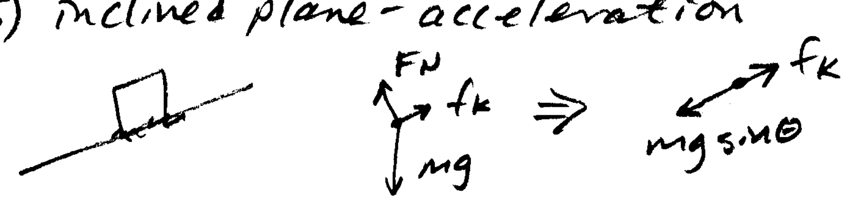
4) Box pulled at angle, no acceleration - Find F needed

$F\cos\theta - \mu_k mg - \mu_k F\sin\theta = 0$

$F(\cos\theta - \mu_k \sin\theta) = \mu_k mg$

$\boxed{F = \dfrac{\mu_k mg}{\cos\theta - \mu_k \sin\theta}}$

5) inclined plane - acceleration



$\Sigma \vec{F} = m\vec{a}$

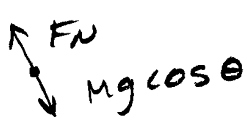
$mg\sin\theta - f_k = ma$

$mg\sin\theta - \mu_k F_N = ma$

$mg\sin\theta - \mu_k(mg\cos\theta) = ma$

$\boxed{a = (\sin\theta - \mu_k \cos\theta)g}$

$\Sigma \vec{F}_y = m\vec{a}^{\,0}$



$F_N - mg\cos\theta = 0$

$F_N = mg\cos\theta$

6) inclined plane - sliding at constant V, $a = 0$

$$\cancel{m}g \sin\theta - \mu_K(\cancel{m}g\cos\theta) = 0$$

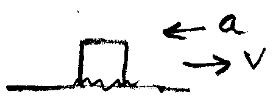
$$\mu_K = \frac{\sin\theta}{\cos\theta}$$

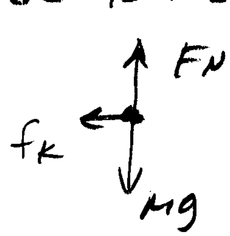
$$\boxed{\mu_K = \tan\theta}$$

7) Inclined plane - angle needed to make object break loose and slide.

$$mg\sin\theta - \mu_s(mg\cos\theta) = 0 \qquad \boxed{\mu_s = \tan\theta}$$

8) Object slide to rest on level.





$$\Sigma \vec{F}_x = m\vec{a}$$

$$f_K = ma$$

$$\mu_K F_N = ma$$

$$\mu_K \cancel{m}g = \cancel{m}a$$

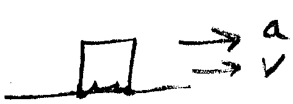
$$\boxed{a = \mu_K g}$$

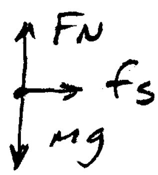
$$\Sigma \vec{F}_y = \cancel{m}\overset{0}{\vec{a}_y}$$

$$F_N - mg = 0$$

$$F_N = mg$$

9) maximum acceleration to move object without slipping.





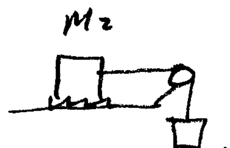
$$\Sigma \vec{F}_x = m\vec{a}$$

$$f_s = ma$$

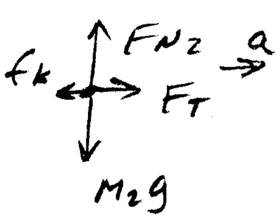
$$\mu_s F_N = ma$$

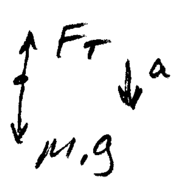
$$\mu_s \cancel{m}g = \cancel{m}a$$

$$\boxed{a = \mu_s g}$$

10) 

mass pulling mass with friction - sliding





$$m_2 g = F_{N_2}$$

$$\Sigma \vec{F}_x = m\vec{a}$$

$$F_T - f_K = m_2 a$$

$$F_T - \mu_K F_{N_2} = m_2 a$$

$$F_T - \mu_K m_2 g = m_2 a$$

$$\Sigma \vec{F} = m\vec{a}$$

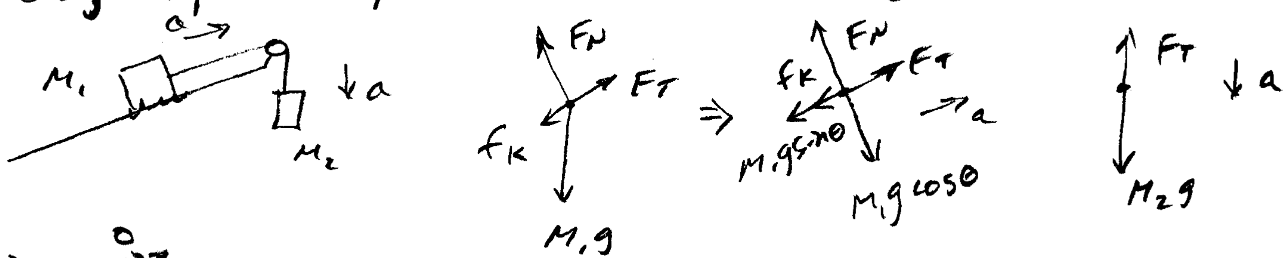
$$m_1 g - F_T = m_1 a$$

add

$$m_1 g - \mu_K m_2 g = m_1 a + m_2 a$$

$$\boxed{a = \frac{m_1 g - \mu_K m_2 g}{m_1 + m_2}}$$

11) object pulled uphill - with friction



$\Sigma \vec{F}_y = \overset{0}{\cancel{m\vec{a}_y}}$

$F_N - M_1 g\cos\theta = 0$

$F_N = M_1 g \cos\theta$

$\Sigma \vec{F} = m\vec{a}$

$F_T - M_1 g\sin\theta - f_K = M_1 a$

$F_T - M_1 g\sin\theta - \mu_K F_N = M_1 a$

$F_T - M_1 g\sin\theta - \mu_K M_1 g\cos\theta = M_1 a$

$\Sigma \vec{F} = m\vec{a}$

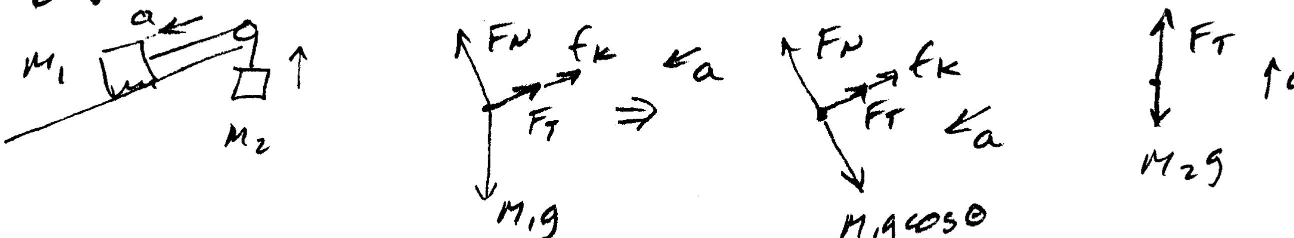
$M_2 g - F_T = M_2 a$

add

$M_2 g - M_1 g\sin\theta - \mu_K M_1 g\cos\theta = M_1 a + M_2 a$

$$\boxed{a = \frac{M_2 g - M_1 g\sin\theta - \mu_K M_1 g\cos\theta}{M_1 + M_2}}$$

12) object with friction accelerating downhill.



$\Sigma \vec{F} = m\vec{a}$

$M_1 g\sin\theta - F_T - f_K = M_1 a$

$M_1 g\sin\theta - F_T - \mu_K F_N = M_1 a$

$M_1 g\sin\theta - F_T - \mu_K (M_1 g\cos\theta) = M_1 a$ add

$\Sigma \vec{F} = m\vec{a}$

$F_T - M_2 g = M_2 a$

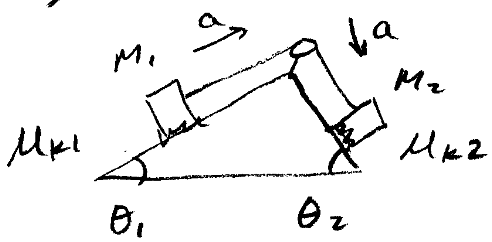
$M_1 g\sin\theta - M_2 g - \mu_K M_1 g\cos\theta = M_1 a + M_2 a$

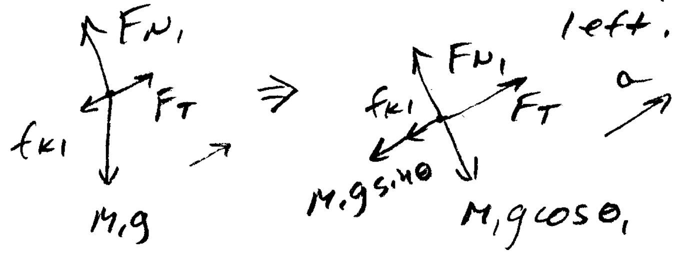
$$\boxed{a = \frac{(M_1 \sin\theta - M_2 - \mu_K M_1\cos\theta)\, g}{M_1 + M_2}}$$

For both 11+12, you need to know which way the objects will accelerate. One way is to do the problem without friction first. Once the direction is established, positive answer for "a" means it will accelerate. Negative value means it will not move.

13) Double hill with friction on both; different angles

acceleration uphill on left.





$\Sigma \vec{F} = m\vec{a}$

$F_T - f_{k1} - m_1 g \sin\theta_1 = m_1 a$

$F_T - \mu_{k1} F_{N1} - m_1 g \sin\theta_1 = m_1 a$

$F_T - \mu_{k1} m_1 g \cos\theta - m_1 g \sin\theta_1 = m_1 a$

$\Sigma \vec{F} = m\vec{a}$

$m_2 g \sin\theta_2 - F_T - f_{k2} = m_2 a$

$m_2 g \sin\theta_2 - F_T - \mu_{k2} F_{N2} = m_2 a$

$m_2 g \sin\theta_2 - F_T - \mu_{k2} m_2 g \cos\theta_2 = m_2 a$

add

$m_2 g \sin\theta_2 - \mu_{k2} m_2 g \cos\theta_2 - \mu_{k1} m_1 g \cos\theta_1 - m_1 g \sin\theta_1 = m_1 a + m_2 a$

$$\boxed{a = \frac{(m_2 \sin\theta_2 - \mu_{k2} m_2 \cos\theta_2 - \mu_{k1} m_1 \cos\theta_1 - m_1 \sin\theta_1)\, g}{m_1 + m_2}}$$

need to determine direction of "a" as on previous page. If "a" is in other direction, some signs will need to change.